The foundation of index calculation

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Foreword

Nowadays, NSIs utilise transaction data, also known as complete data, in increasing amounts in their CPI production. This enforces us to reconsider traditional working practices and index calculation methods.

Following questions, among others, need further discussion: Is there, in the future, need to draw a sample of representative products or outlets, or should we utilise all available data? Is there need to use replacement methods when a product leaves market or solve this with properly selected index number formula? Is there anymore a need for quality assessment when all items are uniquely identified with GTIN-code? Which index number formulas may be applied with complete datasets?

The aim of this paper is to answer to the last question. Alternative index number formulas are summarised so that it renders to take an overall view to the various choices regarding selection of an index number formula.

The result of our work, is a compendium of index number formulas that is presented in this paper. The compendium presents most-known index number formulas that may be used for calculating CPI and HICP[2] index series.

The idea is, that NSIs may use this compendium table to compare pros and cons of an index number formula and to evaluate how these formulas react in typical tests. This list may also be used as a guideline for understanding properties of alternative index number formulas. We see that this facilitates understanding and discussion of alternative approaches.

This summary is a supplement of the document "Circular Error in Price Index Numbers based on Scanner Data. Preliminary Interpretations" (Vartia, Suoperä, Nieminen & Montonen 2018). Also suggestions of Vartia 2018 are used.

Introduction

The objective of the index calculation is to provide a high quality, comparable measures of consumer price inflation (Eurostat, 2017). In order to meet this objective, it is important to ensure compliance with criterion especially when moving from traditional data collection practices towards enhanced procedures where big-data is utilised in large scale.

Statistics Finland started, in 2016, to collect transaction data for the purpose of CPI and HICP compilation. Hence, the first time when scanner data was implemented into production, was in the beginning of 2017 when the pharmaceutical products prices were introduced (Suoperä, Nieminen & Montonen, 2017).

At the moment, weekly data, covering nearly 100 000 single items sold in local retail outlets in Finland, is collected for testing purposes. Data provider is one of the major retail trade group, having 20 regional co-operatives and 1500 outlets all over Finland. This data, so called complete dataset, covers all sold products, their sales value and quantities for years 2014-2016.

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[1] We thank Professor Yrjö Vartia for his comments and consultation concerning the development of new methodologies for processing scanner data.

[2] HICP, Harmonised Indices of Consumer Prices, is used to measure consumer price inflation in the euro area for the purpose of monetary policy. http://ec.europa.eu/eurostat/web/hicp

Before this huge amount of data may be implemented into production, it is vital to test alternative, applicable calculation methods for constructing of an index series. Does some methods give more exact results that the others?

To enable this, we need to separate the main issues in index calculation in order to isolate methodological decisions from one another. Following issues especially relate to index calculation:

1. Selection of an index number formula
2. Tests for identifying the most suitable index number formula for CPI compilation
3. Selection of a construction strategy for index series compilation

To select an index number formula, to distinguish properties of alternative formulas and to identify minimum requirements for an index formula, we need a summary table that puts together suitable index number formulas, their properties and applicable tests. Therefore, we have composed a comparison table 1 that combines main factors describing an index number formula.

In next chapter, we will take a closer look at this table and how it may be used. Various construction strategies, base-, chain- and mixed-strategies, are more closely investigated in paper: Circular Error in Price Index Numbers Based on Scanner Data. Preliminary Interpretations by Vartia, Suoperä, Nieminen and Montonen.

Table of the index number formulas

Table 1, below, is a summary that contains most known index number formulas. The first set is based on formulas using old or new weights: Laspeyres, Geometric Laspeyres and Harmonic Laspeyres use base period weights (i.e. old weights) and Palgrave, Geometric Paasche and Paasche instead use comparison period weights (i.e. new weights). We call these index number formulas as *basic formulas*.

Next set is Jevons and Lowe formulas that are mainly used for calculating price change when item-specific weight information is not available. This has been common practice especially when calculating price change for commodities below an elementary aggregate. Jevons is presented in the table as unweighted, where weight is 1/n, and weighted format. Also FalseDrobisch and FalseFisher[3] are included to this group.

The final set of index number formulas contains eleven (11) index number formulas such as Fisher, Stuvel, Törnqvist, Montgomery-Vartia (Vartia 1). We call these index number formulas as *excellent formulas*. This group includes superlative- and other type index number formulas whose bias is zero (0).

In the columns, first set of columns from one to seven, present index number formula properties while second set, columns 8-12, shows tests for identifying the most suitable index formula for CPI compilation.

[3] Drobisch index is the arithmetic mean of Laspeyres and Paasche indices (ILO 2004, p.348). We call the arithmetic mean of Laspeyres and Palgrave indices as FalseDrobisch and the geometric mean of Laspeyres and Palgrave indices as FalseFisher.

Table 1. The foundation of an index calculation:
Comparison of most typical index number formulas, and test-methods for identification of good index number formula

	Old weights	New weights	Index number formula *	Bias**	Excellent ~ superlative **	Consistent aggregation	Merges null-values ***	Fisher proporti-onality test ****	Time reversal test	Factor reversal test	Chain test	Determinateness test
Laspeyres (La)	X		X	CB		X		X				X
Geom. Laspeyres	X		X	CB		X		X				
Harm. Laspeyres	X		X	CB		X		X				
Palgrave (Pl)		X	X	CB		X		X				X
Geom. Paasche		X	X	CB		X		X				
Paasche		X	X	CB		X		X				X
Jevons (1)			X	CB				X	X		X	
Jevons (w)			??	CB		X		X	X		X	
Lowe (q)			??	CB		X		X	X		X	X
½ (La + Pl) FalseDrobisch	X	X	X	PB				X				X
sqrt(La, Pl) FalseFisher	X	X	X	PB				X				X
Fisher	X	X	X	0	X			X	X	X		X
Stuvel	X	X	X	0	X	X		X	X	X		X
Törnqvist	X	X	X	0	X			X	X			
FA (Törnqvist)	X	X	X	0	X			*	X			
Montgomery-Vartia (Vartia 1)	X	X	X	0	X	X	X	*	X	X		X
Sato-Vartia	X	X	X	0	X		X	X	X	X		X
Marshall-Edgeworth	X	X	X	0	X	X		X	X			X
Vartia-Walsh	X	X	X	0	X	X	X	*	X			X
Diewert	X	X	X	0	X			X	X			

* Index number formula must satisfies CRT, UMT, MUT and PT2 for any n.

** PB = Permanently Biased: Bias(f) exists and ≠ 0.

** CB = Contingently Biased: Bias(f) is not well-defined as a unique real-valued limit, because the limit depends on data.

** 0 = no bias

** Excellent superlative: Bias(f) exists and = 0 (see Vartia & Suoperä (2018, p. 16).

*** Price index does not react at all if the null-values (old or new) are imputed with very small positive values and with *arbitrary* price relatives. All the strong effects go to volume index.

**** Prices and quantities are supposed to change proportionality.

Properties of index number formulas – columns one to seven in table 1

The first set of columns, in the table 1 above, presents properties or features that show how suitable an index number formula is for CPI purposes. Next, we observe each property one by one.

At first, we observe *weight*-information that is needed for calculations with specific index number formula. Table shows the well-known fact: basic formulas use either old or new value shares, while excellent formulas use weight information from both periods, base and comparison.

Next, we evaluate if an *index number formula* meets the requirements. These minimum requirements are set by Pursiainen (2005, p.20) and Vartia (1976, p.57), indicating that a formula must satisfy tests CRT, UMT, MUT and PT2[4] to deserve to be called as an index number formula. Table 1 shows that all other index number formula meet these requirements except weighted Jevons and Lowe. These formulas are rather peculiar as their weights must come outside the compared periods. Especially, satisfaction of CRT and UMT are problematic.

Then, we evaluate which index number formula gives accurate results for small changes or does it bring some kind of *bias* with it. We notice that first nine in the list are *contingently biased (CB).* The size of bias depends on the data in question: it may be small or large. Next two index number formulas, FalseDrobisch and FalseFisher, are *permanently biased (PB)*. This means that these index number formulas are badly biased upwards all the time. Those formulas that have zero (0) in this column are unbiased all the time.

Excellent~superlative-column shows formulas, that are defined in CPI-manual (ILO 2004) as superlative index number formulas or that are excellent[5] (called pseudo-superlative by Diewert 1978) index number formulas. Excellent or pseudo-superlative are quadratic approximations of superlative indices for small changes.

Consistent aggregation-column shows formulas that meet the requirement for exact consistency in aggregation. For example Fisher and Törnqvist index number formulas are not exactly consistent in aggregation, instead are approximately consistent in aggregation. (ILO 2004, p.326). This precision does not meet the requirements set for HICP –series.

Merges null-values- column presents index number formulas that meet the requirement to accept null-values (i.e. new or vanishing commodities). This means that there is no need to impute null-values, because the imputation of new or vanishing commodities with small weights has no effects on the results, see Vartia 2018. Table 1 shows that only three of the listed nineteen formulas meet the requirement.

Next, we take a look at columns 8-12 listing other tests for identifying good index number formulas. This topic is treated in the next chapter.

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[4] Commodity reversal test (CRT): The price index should remain unchanged if the ordering of the commodities is changed (ILO 2004, p.294).
Unit of measurement test (UMT): The price index does not change if the units of measurement for each commodity are changed (ILO 2004, p.294). ILO calls this test the commensurability test.
Money unit test (MUT): The money unit has no effect on the index number. (Vartia 1976, p.57).
Weak proportionality test (PT2): See next chapter.
[5] This is the second best class of indices in Fisher 1922 after superlative.

Test-methods for identification of good index number formula

In this chapter, we formulate and explain tests that are presented in the columns 8-12. We use the same index number definition as Pursiainen. Thus, *an index number formula* is defined as a sequence of functions

$$(f_n)_{n\in\mathbb{N}},\ \ f_n\colon (\mathbb{R}^n_{++})^4 \to \mathbb{R}_{++},$$

where f_n satisfies CRT, UMT, MUT and PT2 for every n.

A price index for n commodities is denoted by $f_n\begin{pmatrix} p^1 & q^1 \\ p^0 & q^0 \end{pmatrix}$,[6] where p^1, q^1 are the period 1 or comparison period prices and quantities respectively and p^0, q^0 are the period 0 or base period prices and quantities (Pursiainen 2005, p.20).

Proportionality test

An index number formula satisfies *Fisher proportionality test* if

$$\forall\ k > 0\colon \forall\ (p^1, p^0, q^1, q^0)\colon p^1 = kp^0\colon\ \ f_n\begin{pmatrix} p^1 & q^1 \\ p^0 & q^0 \end{pmatrix} = k.$$

That is, if all individual prices change in the same proportion from period 0 to period 1, the price index should be equal to the common factor of proportionality.

An index number formula satisfies *weak proportionality test* (PT2) if

$$\forall\ k > 0\colon \forall\ l > 0\colon \forall\ (p^1, p^0, q^1, q^0)\colon p^1 = kp^0\ and\ q^1 = lq^0\colon\ \ f_n\begin{pmatrix} p^1 & q^1 \\ p^0 & q^0 \end{pmatrix} = k.$$

In this weak proportionality test prices and quantities are supposed to change proportionality at the same time and in such a situation value shares remain constant (Vartia 1976a, p.12). Factor antithesis of Törnqvist (FA Törnqvist)[7], Montgomery-Vartia and Vartia-Walsh are the only formulas that satisfy weak proportionality test (PT2) but not Fisher proportionality. Because of their excellence, they satisfy Fisher proportionality very accurately. All other formulas satisfy both tests.

Time reversal test

An index number formula satisfies *the time reversal test* if

$$\forall\ (p^1, p^0, q^1, q^0)\colon\ \ f_n\begin{pmatrix} p^1 & q^1 \\ p^0 & q^0 \end{pmatrix} = 1/f_n\begin{pmatrix} p^0 & q^0 \\ p^1 & q^1 \end{pmatrix}.$$

That is, if the period 0 and period 1 price and quantity data are interchanged, then this new price index is equal to the reciprocal of the original price index (ILO 2004, p.267). Table 1 shows that basic index formulas do not satisfy this test, because of their temporal asymmetry. This is a serious drawback and makes them contingently biased.

Factor reversal test

An index number formula satisfies *the factor reversal test* if

$$\forall\ (p^1, p^0, q^1, q^0)\colon\ \ f_n\begin{pmatrix} p^1 & q^1 \\ p^0 & q^0 \end{pmatrix} f_n\begin{pmatrix} q^1 & p^1 \\ q^0 & p^0 \end{pmatrix} = V^1/V^0.$$

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[6] Alternative notation is $P_0^1 = f_n(p^1, p^0, q^1, q^0)$.

[7] The factor antithesis of Törnqvist arises by first using the same formula for quantities, tor(q), and then returning back to the price space by calculating $(V^1/V^0)/tor(q)$ (Vartia & Suoperä (2018), p.10).

That is, the product of the price index and the quantity index calculated by same index number formula ought to equal the value ratio (ILO 2004, p.297). Only four listed index number formulas satisfy the factor reversal test.

Chain test

An index number formula satisfies *the chain or circular test* (CT) if

$$\forall\,(p^2,p^1,p^0,q^2,q^1,q^0):\ f_n\begin{pmatrix}p^1 & q^1\\ p^0 & q^0\end{pmatrix} f_n\begin{pmatrix}p^2 & q^2\\ p^1 & q^1\end{pmatrix} = f_n\begin{pmatrix}p^2 & q^2\\ p^0 & q^0\end{pmatrix}.$$

That is, the product of the two price-links $0 \rightarrow 1$ and $1 \rightarrow 2$ should equal the price link $0 \rightarrow 2$. Time reversal test TRT is a special case of this (set 2=0). Every formula failing TRT fails also CT.

If the equation does not hold, the chained indices are said to suffer from *chain drift* (ILO 2004, p.445). Only Jevons and Lowe satisfy this test, because they use a fixed set of weights. It is demonstrated in our paper, that excellent index number formulas satisfy CT approximately if the same ideal demand theory describes all the links.

Determinateness test

The determinateness test means that, if any single price p^k or quantity $q^k, k = 0{,}1$ tends to zero, then the price index should not tend to zero or infinity (ILO 2004, p.309, Vartia 1976, p.82). For example, Jevons and Törnqvist do not satisfy this test.

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Annex 1. The alternative index number formulas

The formulas, in the table 1, are defined in following documents:

Laspeyres (La): See ILO (2004), p.3.

Geom. Laspeyres: See ILO (2004), p.163.

Harm. Laspeyres: See ILO (2004), p.303.

Palgrave (Pl): See ILO (2004), p.302.

Geom. Paasche: See ILO (2004), p.301.

Paasche: See ILO (2004), p.3.

Jevons (1): See ILO (2004), p.361.

Jevons (w): See ILO (2004), p.282.

Lowe (q): See Vartia (1976), p.60.

½ (La + Pl) = FalseDrobisch: See footnote 3.

sqrt(La, Pl) = FalseFisher: See footnote 3.

Fisher: See ILO (2004), p.6.

Stuvel: See Vartia (1976), p.203.

Törnqvist: See ILO (2004), p.6.

FA (Törnqvist): See footnote 7.

Montgomery-Vartia (Vartia 1): See Vartia (1976), p.124.

Sato-Vartia: See Vartia (1976), p.128.

Marshall-Edgeworth: See ILO (2004), p.268.

Vartia-Walsh: See Vartia & Suoperä (2018), p.29.

Diewert: See Vartia & Suoperä (2018), p.20.