Contrasting GEKS strategies: compilation of CPI with fixed-base and chain-linking with scanner data

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Abstract

There is a consensus, that multilateral strategies are the preferred choice for compiling consumer price indices (CPI), if *complete* scanner datasets are available. Yet, statistical offices need to carefully consider the practical issues that arise with scanner data and adopt strategies that are feasible and tractable. Multilateral GEKS, for instance, can be adopted in number of ways. This article explores the chain drift, arising from the rolling window GEKS strategies, and compares them to strategies that adopt fixed bases with 13 period windows. The comparison is done empirically using stable data. The results show, as expected, that the rolling window chains may generate a serious drift, contingent on the commodity categories, while the alternative fixed-base strategies do not generate a drift, are consistent with the widely recommended multilateral GEKS, and are easier to compile. The results underline, that the new commodities are a minor problem compared to chain drift caused by the chain strategy of the GEKS.

Keywords: Chain Drift, GEKS strategies, Multilateral Methods, Consumer Price Index, Scanner Data

1 Introduction

From an index theorist's perspective, one of the greatest benefits of digitalization has been the growing widespread availability of more complete datasets which have enabled the use of more sophisticated index formulae. Naturally, empirical literature comparing and measuring the different properties of such indexes have emerged. Thus far, the chain strategy has been the main focus (see Ivancic, Fox and Diewert (2011); Nygaard (2010); de Haan and van der Grient (2010, 2011); Diewert and Fox (2017)). This paper provides complementary evidence on the usefulness of adopting fixed bases in a multilateral strategy. Our primary focus is to compare the chain-linking and fixed-base strategies and in doing so, estimate the amount of drift arising from chain-linking. These comparisons are done with the Walsh's multi period identity test (MPIT).

The second objective is related to Pursiainen (2005, pp 2): '*The subaggregates and the overall aggregate are usually computed using the same method, for example the same index number formula. This presents a two-faceted consistency problem. First, is it possible to obtain the overall aggregate by using only the subaggregates? Second, if this is possible, can the overall aggregate be derived from the subaggregates using the same method that was used to calculate the subaggregates?*' We apply this idea herein. That is, we evaluate the effects of changes in the aggregation rules. As mentioned, there are several ways of constructing the GEKS. One possible way is to compute sub-aggregates - necessary bilateral building blocks - by the excellent index number formulae (i.e., Fisher, Törnqvist…) and then aggregate them together using the Jevons formula (Ivancic et al., 2011) (i.e., unweighted geometric average). Another way is to use the same exact formula in all aggregation steps. Hence, our paper presents a fourfold problem. We adopt two strategies, fixed bases and chain-linking, with two aggregation rules, non-weighted and weighted. This approach provides a useful case-study for statistical offices interested in adopting scanner data sets and multilateral strategies.

As pointed out in ILO (2004, pp. 407), the main question of the choice of multilateral strategy is: Should we prefer fixed-base, or chain-linking strategy? Our empirical results confirm the observations in Vartia, Suoperä, Nieminen and Montonen (2018, 2021), that using the Ivancic, Diewert & Fox GEKS strategy will generate chain drift contingently on data. That drift arises contingently, is a serious problem in official production of CPI. To demonstrate the problem, we separate the data into three categories based on the severity of the drift. In the first category of items, the chain drift appears '*harmless*' for some commodity groups independently of the problem of new and disappearing commodities. In the second category, the chain drift is harmless for some time periods but emerges and cumulates over time. In the third category, the chain drift arises almost all the time periods and cumulates seriously.

The study is arranged as follows: In chapter 2 we present the data, notations and the index number formulae. In chapter 3 we define a multilateral fixed-base and chain-linking strategies. In chapter 4 we present empirical results and chapter 5 concludes.

2 Test Data, Notations and Index Number Formulae

2.1 Description of Test Data

As in Vartia, Suoperä, Nieminen and Montonen, 2018, 2019, we use the scanner data that contains information on three chained shops: 'hypermarkets', 'supermarkets' and 'small shops'. The data set contains monthly observations from years 2016 – 2020 collected from five regions. The data is classified by region, chained shop type, coicop6, kat9 commodity group. The analyzed commodity groups, at coicop6 level, are: 'Beef', 'Pork', 'Broiler meat', 'Other meat products', 'Fish' and 'Milk powder'. These categories include about 10000 – 15000 commodities that are comparable in quality in all time periods.

2.2 Notation

Our notation for the index number calculations follows the convention in Vartia and Suoperä (2017, 2018); Vartia, Suoperä, Nieminen and Montonen (2018) and Suoperä and Auno (2021), as explained below

Commodities: $a_1, a_2, \dots, a_{n_t}$ in period *t*.
Time periods: *t* = 0, 1, 2, … are the compared months
Quantity: $q_i^t = q_{it}$ is the quantity of a_i in period *t*.
Value: $v_i^t = v_{it} = q_{it}p_{it}$ is the value of a_i in period *t*.
Unit value: $p_i^t = v_i^t/q_i^t$ or $p_{it} = v_{it}/q_{it}$ is the unit price of a_i in period *t*.
Total value: $V^t = \sum_i v_i^t = \sum_i v_{it}$ is the total value of all the commodities in period *t*.
Total quantity: $Q^t = \sum_i q_i^t = \sum_i q_{it}$ is the total quantity of all the commodities in period *t*.
Price relatives: $p_i^{t/0} = p_{it}/p_{i0}$ is the price relative of a_i from period 0 to *t*.
Quantity relatives: $q_i^{t/0} = q_{it}/q_{i0}$is the quantity relative of a_i from period 0 to *t*.
Value relatives: $v_i^{t/0} = v_{it}/v_{i0}$ is the value relative of a_i from period 0 to *t*.
Value shares: $w_{it} = v_{it}/\sum_i v_{it}$ is the value share of a_i in period *t*.

The above notations are modified to accommodate the analysis of multilateral fixed-base and chain-linking strategies. Corresponding n_t-vectors presented as bolt for the above symbols without sub-index are:

$$(\boldsymbol{p}_t,\ \boldsymbol{q}_{t\,,}\boldsymbol{v}_t,\ \boldsymbol{w}_t,\ \boldsymbol{p}^{t/0},\ \boldsymbol{q}^{t/0},\ \boldsymbol{v}^{t/0})$$

We assume that all prices and quantities are strictly positive (contain no zeros). This implies that all values, prices, quantities, value relatives and value shares are also well-defined and strictly positive.

2.3 Index Number Formulae

We make use of a simple bilateral notation (see Vartia, 2010; Nieminen & Montonen, 2018) as:

$$(p^k,\ q^k, p^t,\ q^t) \rightarrow\ P_n(p^k,\ q^k, p^t,\ q^t) = {P_n}^{t/k}$$

where the price index ${P_n}^{t/k}$is based on price-links from a period *k* to the period *t*. The bilateral indices for properly chosen *k*, *t* =1,… ,T are the basic building blocks for the multilateral strategies. We select the entire 13-period (i.e., T = 13 month) window to define the two different multilateral strategies, which are described later on in chapter 3.

We use six formulae: Stuvel (*S*), Montgomery-Vartia (*MV*), Törnqvist (*T*), Fisher (*F*), Sato-Vartia (*SV*) and Walsh-Vartia (*W*). The literature calls these index number formulae as excellent, but three of them (i.e. *F*, *T*, *W*) are also Diewert-superlative (Diewert, 1976, 1978). All the mentioned formulae satisfy the time reversal test (*TR*). In this study, all index number formulae are based on their logarithmic representation (see Suoperä, Nieminen, Montonen and Markkanen, 2021 pp.5-6) for arbitrary time periods (*k*, *t*). The logarithmic representations are desired because they transform the quite difficult multiplicative GEKS-strategies into simpler additive form. Index numbers in logarithmic form are presented as

(1) $$log\left(P_f^{t/k}\right) = \sum_i w_{i,f} \cdot log\left(p_i^t/p_i^k\right),$$

where weights of the formula are presented in the following table.

Table 1: Observation level weights for index number formulae in logarithmic form (i.e. for the equation (1)).

Index number formula (L means here the logarithmic mean, Vartia, 1976)	
Törnqvist, f = T	$w_{i,f} = \bar{w}_{i,T} = 0.5 \cdot (w_i^k + w_i^t)$
Sato-Vartia, f = SV (Vartia, 1976, pp. 130-131)	$w_{i,f} = \bar{w}_{i,SV} = \frac{L(w_i^t, w_i^k)}{\sum L(w_i^t, w_i^k)}$
Montgomery-Vartia, f = MV (Vartia, 1976, pp. 124-125)	$w_{i,f} = \bar{w}_{i,MV} = L(p^t q^t, p^k q^k)$
Fisher, f = F (Vartia, 1976, pp.128)	$w_{i,f} = \bar{w}_{i,F} = 0.5 \cdot \left(L(p^t q^k, p^k q^k) + L(p^t q^t, p^k q^t)\right)$
Walsh, f = W	$w_{i,f} = \bar{w}_{i,W} = \left(w_i^k \cdot w_i^t\right)^{1/2}$
Stuvel, f = S (Pursiainen, 2005, pp. 88)	$w_{i,f} = \bar{w}_{i,S} = L(p^t \bar{q}, p^k \bar{q})$

The equation (1) forms the basic building block for any multilateral method including the base and chain strategies. For more fundamental analysis of these formulae, see Vartia & Suoperä (2018).

3 Multilateral strategy

Multilateral, or *multi bilateral* strategies are based on seminal papers of Gini (1931); Eltetö & Köves (1964) and Szulc (1964). They develop the GEKS method, where price and output comparisons are made across economic entities, such as countries. The GEKS method was applied for time periods by Balk (1981) and more recently by Ivancic, Diewert and Fox (2011), marked as *IDF*. Multilateral strategies are an attractive method due to their ability to control for chain drift bias, although researchers do not agree on the best approach to updating the series, when new data comes available, and on the best multilateral method to use. We compare two types of multilateral strategies, with fixed bases, and with rolling windows. These strategies are computed with two different aggregation rules. The first one allows no change of aggregation rule for separate bilateral building blocks of the GEKS and overall aggregation of them (i.e., use the same formula is used, either *F*, *T*, *MV*… for all steps of aggregation). In the second one, we allow a change of the aggregation rule similarly as in Ivancic, Diewert & Fox GEKS strategy (i.e., use *F, T, MV*… for the bilateral building blocks of the GEKS, but change the aggregation rule into unweighted geometric average to get the overall index). In this way, we provide a test for the consequences of changing the aggregation rule.

To summarize, our research problem is fourfold and encompasses two strategies and two aggregation rules. As is standard in the literature and with monthly data, we use 13 time periods for our tests. In the fixed-base strategy, the first 12 periods (months) come from the previous year and the 13th comes from a current year. In the chain-linked strategy we use the familiar 13-period rolling GEKS as in *IDF*. The fixed-base strategy satisfies the *transitivity/chain property* and the *Walsh´s Multi Period Identity Test* (*MPIT*) all the time, but the 13-period rolling GEKS only for a given fixed 13-period window. In other words, for a given 13 period window all price comparisons are based on the base strategy, where each month of the window in turn is the base. The base strategy trivially satisfies the transitivity property (for useful discussions about these properties, see Vartia, Suoperä, Nieminen and Montonen 2018, 2021, pp. 3 - 5). Instead, the construction strategy of index series for 13-period rolling GEKS is based on the chain strategy and according to Fisher (1922, pp. 271) any index number formula (equal weighted Jevons excluded, or $Q_f^{t/k} = 1$, for all k, t) does not satisfy the *transitivity/chain property and of course the Walsh's MPIT*. For any chain strategy drift necessarily happens.

3.1 The 13-period Chain Strategy of the Rolling GEKS

Our test data starts 2016 January and ends 2020 September. The data includes 57 months, t = 1, …, 57. The basic idea of the rolling GEKS follows the idea of the moving averages in time series analysis. In practice, this means that the overall set of the periods is divided into the following 13-period splices – the first splice includes first 13 time periods (t = 1, 2, …, 13), the second splice (t = 2, 3, …, 14) time periods etc. For our 57 time periods, we get 45 splices, or windows, of GEKS. These windows include the knowledge of prices, quantities and values of commodities consumed in each time period. Diewert and Fox crystallize the idea how the bilateral building blocks are be formed: '*Now pick one month (say month k) in this augmented year as the base month and construct Fisher price indexes for all 13 months relative to this base month*' (Diewert and Fox, 2018, pp. 9). Using our notations, we get the following set of the price index numbers $\left\{P_f^{1/t}, P_f^{2/t}, \dots, P_f^{12/t}, P_f^{13/t}\right\}$, where t = 1, 2, …., 13 and subindex f stands for index number formula, in our case f = S, T, MV, SV, W, F (see above eq. (1) and Table 1). As is necessary, all the index number formulae satisfy the time reversal test (*TR*).

In practice, we construct index series starting from 2016 December, so that P(2016.12) =1. Next, we calculate price index numbers only for two last periods for all windows. With these index numbers we construct index series following the principle described in *IDF*. At first, however, our formulation of the 13-period rolling GEKS follows the principle suggested in Pursiainen (2005, pp. 2) – and apply the same method of aggregation in each step. For the first window (i.e., for t = 1, 2, …., 13) we get (see Vartia, Suoperä, Nieminen and Montonen 2018, 2021, Theorem 1, pp. 4)

(2a) $$P_f^{13/12} = \exp\left\{\sum_{t=1}^{13} w_{t,13,f} \cdot log\left(P_f^{13/t}\right) - \sum_{t=1}^{13} w_{t,12,f} \cdot log\left(P_f^{12/t}\right)\right\},$$

where $w_{t,13,f}$ and $w_{t,12,f}$ are weights for index number formulae f = S, T, MV, SV, W, F in second step of aggregation. The second window includes time periods t = 2, 3, …., 14 and we get

(2b) $$P_f^{14/13} = \exp\left\{\sum_{t=2}^{14} w_{t,14,f} \cdot log\left(P_f^{14/t}\right) - \sum_{t=2}^{14} w_{t,13,f} \cdot log\left(P_f^{13/t}\right)\right\} \text{ and}$$

for third window

(2c) $$P_f^{15/14} = \exp\left\{\sum_{t=3}^{15} w_{t,15,f} \cdot log\left(P_f^{15/t}\right) - \sum_{t=3}^{15} w_{t,14,f} \cdot log\left(P_f^{14/t}\right)\right\}$$

The same process continues up to the last window no. 45 which contains time periods t = 45, 46, …., 57. The index series are constructed as in *IDF*, that is

(2d) $$\begin{aligned} P_f^{12/12} &= 1 \\ P_f^{13/12} &= P_f^{12/12} * P_f^{13/12} \\ P_f^{14/12} &= P_f^{12/12} * P_f^{13/12} * P_f^{14/13} \\ P_f^{15/12} &= P_f^{12/12} * P_f^{13/12} * P_f^{14/13} * P_f^{15/14} \\ &\vdots \\ P_f^{57/12} &= P_f^{12/12} * P_f^{13/12} * \cdots * P_f^{56/55} * P_f^{57/56} \end{aligned}$$

and this is the first multilateral strategy for 13-period rolling GEKS. The equation (2d) clearly demonstrates that the method is a chain strategy.

Next, we change the aggregation rule in the second step, for example from Törnqvist, into the unweighted geometric average of 13 bilateral indices. Change of weighting changes above method (i.e., eq. (2a) - (2d)) precisely to the method of *IDF*. For the first window we get (now in logarithmic form)

(3) $$P_{f,IDF}^{13/12} = \exp\left\{1/13 \cdot \left(\sum_{t=1}^{13} log\left(P_f^{13/t}\right) - \sum_{t=1}^{13} log\left(P_f^{12/t}\right)\right)\right\},$$

All other windows in *IDF*-method are analyzed similarly as in (2a) - (2d)) but replacing index weights by 1/13. This is our second 13-period rolling GEKS. Both 13-period rolling GEKS-methods are based on the familiar *chain-linking strategy*. They satisfy the *transitivity property* and the *Walsh´s Multi Period Identity Test* (*MPIT* separately in each window. This does not hold for rolling windows, which necessarily implies some chain drift. (see Vartia, Suoperä, Nieminen and Montonen, 2018, pp 3 - 5).

3.2 The 13-period Fixed-Base Strategy of the GEKS

The fixed-base strategy of GEKS is a relatively unknown method for most index number experts. This section defines two of them. The time periods are selected as follows: the first set of time periods includes the previous year (i.e., 12 months) and thirteenth period is January of a current year. The idea is to form price-links from each month of previous year into January (i.e., period 13), that is $\{1,2,\ldots,12\} \rightarrow \{13\}$. In the second set, January of a current year is replaced by February, that is $\{1,2,\ldots,12\} \rightarrow \{14\}$. This process continues up to December of a current year $\{1,2,\ldots,12\} \rightarrow \{24\}$, at which point, the base period is changed as $\{13,14,\ldots 24\} \rightarrow \{25\}$. The base changes from $\{1,2,\ldots,12\}$ into $\{13,14,\ldots 24\}$ or, in our data, from year 2016 to 2017. The index series based on different bases are linked together easily always in December. Using the notations used in the index compilation, we get an index number set $\left\{P_f^{13/t}, P_f^{14/t}, \ldots, P_f^{23/t}, P_f^{24/t}\right\}$, where t = 1, 2, …., 12 and subindex f stands for index number formula, which are here f = *S*, *T*, *MV*, *SV*, *W*, *F* (see above eq. (1) and Table 1). The overall index using the same formula also in the second aggregation step becomes (subindex *B* denotes that we are dealing with base strategy)

(4a) $$P_{f,B}^{13/0} = \exp\left\{\sum_{t=1}^{12} w_{t,13,f} \cdot log\left(P_f^{13/t}\right)\right\},$$

where 0 stands for all months from previous year. For February (14th time period) we get

(4b) $$P_{f,B}^{14/0} = \exp\left\{\sum_{t=1}^{12} w_{t,14,f} \cdot log\left(P_f^{14/t}\right)\right\} \text{ and up to December}$$
$$\vdots$$
$$P_{f,B}^{24/0} = \exp\left\{\sum_{t=1}^{12} w_{t,24,f} \cdot log\left(P_f^{24/t}\right)\right\}.$$

The price change for example from December to January is simply

(4c) $$P_{f,B}^{13/12} = P_{f,B}^{13/0} / P_{f,B}^{12/0} \text{ and more generally, } P_{f,B}^{k/j} = P_{f,B}^{k/0} / P_{f,B}^{j/0}, \text{ for } k, j = 13, \ldots, 24$$

because all our index number formulae satisfy the time reversal test (TR). When the 25'th period emerges, we change the base and the proses proceed similarly as in equation (4a) to (4c). We form price-links $\{13,14,\ldots 24\} \rightarrow \{25\}$; $\{13,14,\ldots 24\} \rightarrow \{26\}$ etc. This base strategy of the GEKS satisfies the *transitivity/chain property* and the *Walsh´s Multi Period Identity Test* (*MPIT*) over time and is free of the chain drift. This is our benchmark strategy for which the MPIT is based on.

The second fixed-base strategy of the GEKS is derived analogously as in the previous section, the weights of the index number formula are replaced by 1/12. Let *uB* denote the unweighted fixed-base strategy

(5) $$P_{f,uB}^{13/0} = \exp\left\{1/12 \cdot \sum_{t=1}^{12} log\left(P_{f,uB}^{13/t}\right)\right\}$$

The process continues for unweighted base GEKS (5) similarly as in eq. (4b) and (4c).

We have now two proper GEKS strategies – one with chain-linking, and one with fixed bases – which are derived using the same aggregation rule in different aggregation steps. This means that for these two strategies, we do not change the index number formula for different aggregation steps. This is a natural choice for most aggregation experts. For the two other strategies, the monthly differences in consumption patterns are ignored/removed by taking the unweighted geometric average (see eq. (3) and (5) for the weights of the rolling window GEKS and fixed base, respectively). Table 2 below summarizes the discussion above.

Table 2: Summary of strategies.

Strategy\Aggregation rule	No change of aggregation rule	Change of aggregation rule
Base strategy	Strategy 1 (*S*, *T*, *MV*, *SV*, *W*, *F* in all steps)	Strategy 2 (*S*, *T*, *MV*, *SV*, *W*, *F* replaced by 'Jevons' in second step)
Chain strategy	Strategy 4 (*S*, *T*, *MV*, *SV*, *W*, *F* in all steps)	Strategy 3 (*S*, *T*, *MV*, *SV*, *W*, *F* replaced by 'Jevons' in second step)

The empirical results shed light on the differences between strategies, and on the differences between aggregation rules, as detailed in the table 2 above. We believe this is useful for the practical compilation of CPIs.

4 Empirical Results

This study analyzes six excellent index number formulae, including three Diewert-superlative formulae (i.e., *T*, *F* and *W*). For example, Vartia & Suoperä, 2018 shows that these six formulae are practically equal. Figures 1 - 4 tell the same story for most regions, coicop6 groups, and strategies 1-4.

Figure 1: Index series for *S*, *T*, *MV*, *SV*, *W*, *F* for strategy 1, in region Uusimaa and coicop6 = 'Pork' from December 2016 to September 2020.

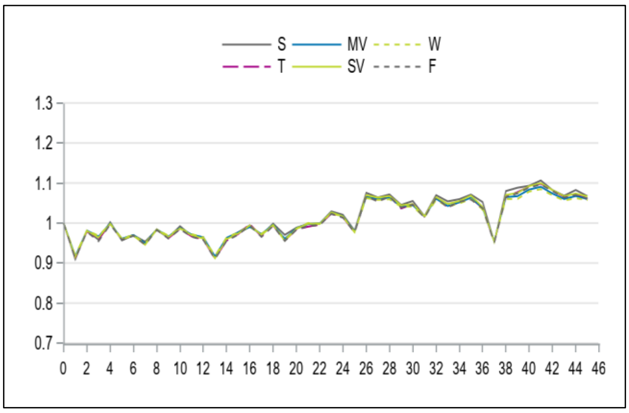


Figure 2: Index series *S*, *T*, *MV*, *SV*, *W*, *F* for strategy 2, in region Etelä-Suomi and coicop6 = 'Pork' from December 2016 to September 2020.

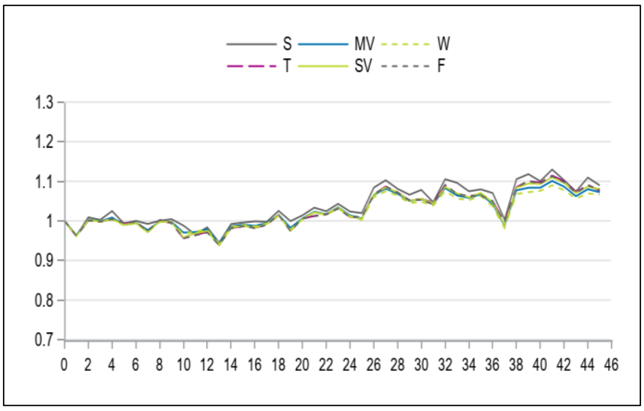


Figure 3: Index series for *S*, *T*, *MV*, *SV*, *W*, *F* for strategy 3 in region Itä-Suomi and coicop6 = 'Pork' from December 2016 to September 2020.

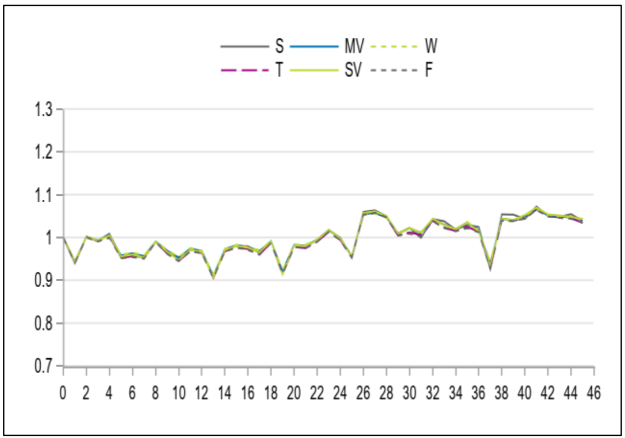
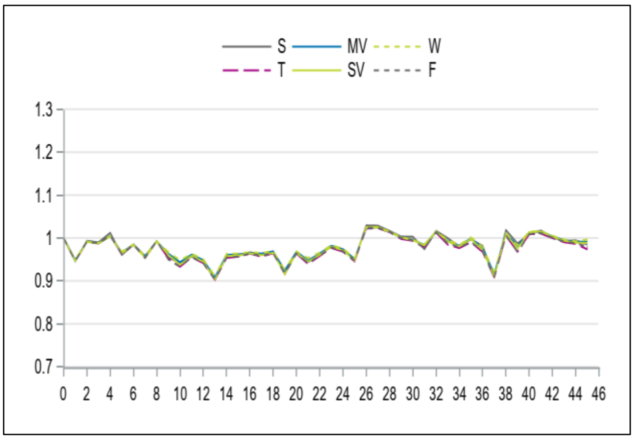


Figure 4: Index series *S*, *T*, *MV*, *SV*, *W*, *F* for strategy 4 in region Länsi-Suomi and coicop6 = 'Pork' from December 2016 to September 2020.



In most regions and commodity groups (here coicop6) index number formulae behave as in Figures 1 to 4 'hand-in-hand', although Fisher and Stuvel are a slight exception – for some regions and commodity groups they deviate slightly from others. The main conclusion is however that the selection of formula does not play any role in generating the drift. Later on, in section 4.2, we will show that the choice of strategy indeed plays a major role.

4.1 The MPIT

We apply the Walsh's *MPIT* for index numbers *f* satisfying the time reversal test (*TR*). The *MPIT* is done stepwise as follows

(6)
$$\begin{aligned} &P_f^{2/1} \cdot P_f^{1/2} = 1 \\ &P_f^{2/1} \cdot P_f^{3/2} \cdot P_f^{1/3} = 1 \\ &\qquad\vdots \\ &P_f^{2/1} \cdot P_f^{3/2} \cdot \ldots \cdot P_f^{t/(t-1)} \cdot P_f^{1/t} = 1 \end{aligned}$$

where *f* = *S*, *T*, *MV*, *SV*, *W*, *F*. At every test step the Walsh's *MPIT* asks whether the product of all price changes included in the test step equals direct price change from $t \rightarrow 1$. If the tests differ from unity the chain drift occurs. The strategies with fixed-bases derived in eq. (4a) – (4c) and (5) satisfy the *MPIT* trivially (replace price changes in (6) by price changes for example in (4c) in each test step, see Vartia, Suoperä, Nieminen and Montonen, 2018, pp 3 - 5). In practice, we can therefore compare the fixed-base GEKS for example to rolling window GEKS of *IDF*

$$\begin{aligned} &P_{f,IDF}^{2/1} \cdot P_{f,B}^{1/2} = 1 \qquad ? \\ &P_{f,IDF}^{2/1} \cdot P_{f,IDF}^{3/2} \cdot P_{f,B}^{1/3} = 1 \qquad ? \\ &\qquad\vdots \\ &P_{f,IDF}^{2/1} \cdot P_{f,IDF}^{3/2} \cdot \ldots \cdot P_{f,IDF}^{t/(t-1)} \cdot P_{f,B}^{1/t} = 1 \qquad ? \end{aligned}$$

The subindex *B* refers to fixed-base strategy (i.e., eq. (4) and (5)) and subindex *IDF* to IDF. The question mark asks whether the test equals unity. Some may now ask: 'Why do we test the multilateral rolling GEKS by the *MPIT*, because according to Ivancic, Diewert and Fox (2011) it satisfies the test'. But this is not true for the 13-period ***rolling*** GEKS, although the test is satisfied for any given 13-period window. The proofs for single window are given in Vartia, Suoperä, Nieminen and Montonen (2018, Chapter 3). The index series for the rolling GEKS are based on the chain strategy. Simply put, the multilateral rolling GEKS uses *multi bilateral price-links* that are linked together using the principle of the chain strategy (see eq. (2d)), and, in any such strategy the chain drift necessary occurs (Fisher, 1922, pp. 271).

4.1.1 Harmless Chain Drift

Next, we show the contingent nature of the 13-period rolling GEKS. This means that 'here-and-there' the indices will include chain drift. We start from the harmless commodity groups (i.e., region*coicop6-group). We use Törnqvist (*T*) formula to show the results, but any of the formulae explored in our study will show very similar results. In the figures below, the left side presents the index series for our four strategies and the right side displays the corresponding *MPIT*'s. We define chain drift as 'Harmless' if the test deviate from unity at most ± 0.5 %. Harmless situations cover 30% of all the *MPIT*s.

Figure 5: Törnqvist for strategies 1 to 4 in region Uusimaa coicop6 = 'Broiler meat' from December 2016 to September 2020.

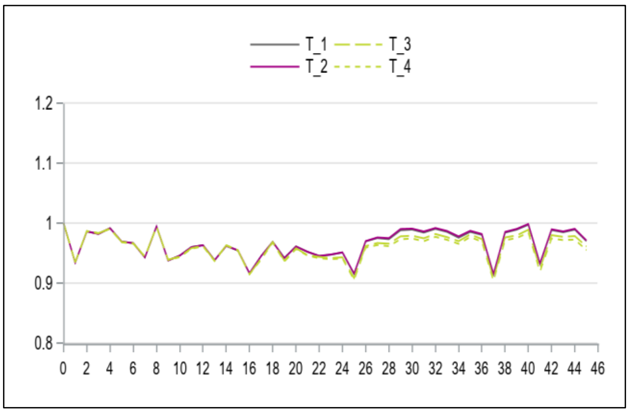


Figure 6: The corresponding MPIT's.

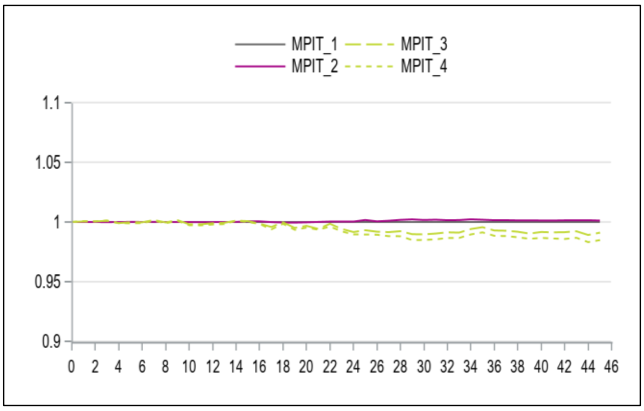


Figure 7: Törnqvist for strategies 1 to 4 in region Itä-Suomi coicop6 = 'Milk powder' from December 2016 to September 2020.

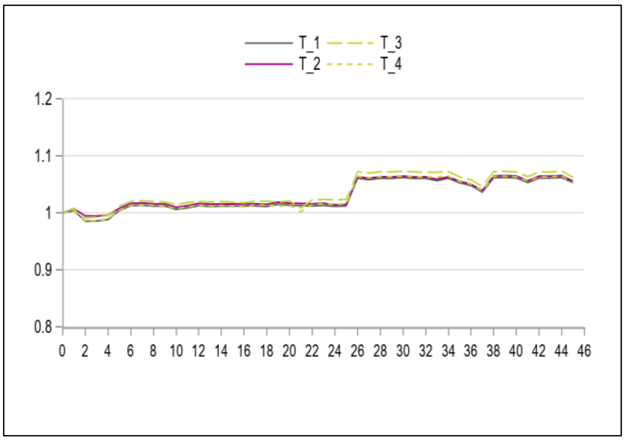


Figure 8: The corresponding MPIT's.

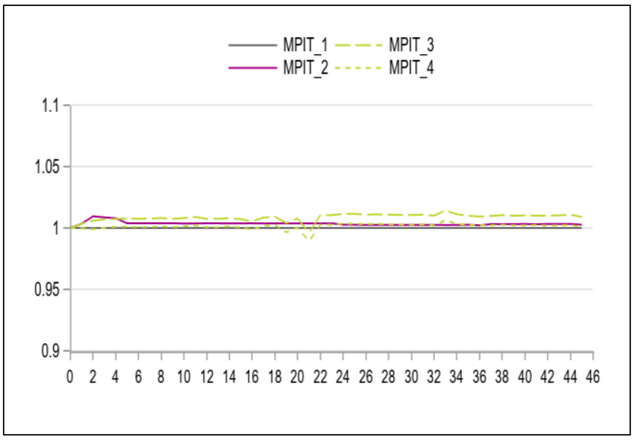


Figure 9: Ratio of consumption values and number commodities in region Uusimaa and coicop6 = 'Broiler meat' from December 2016 to September 2020.

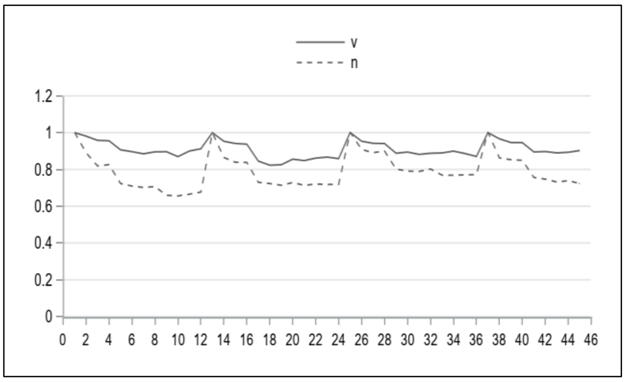
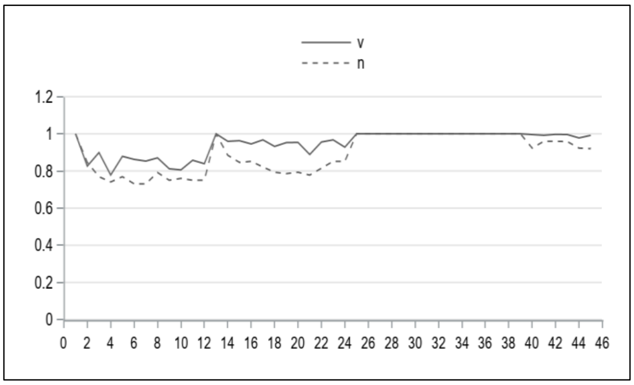


Figure 10: Ratio of consumption values and number commodities in region Itä-Suomi and oicop6 = 'Milk powder' from December 2016 to September 2020.



Only 2 out of 30 (i.e., 5 regions and 6 coicop6) commodity groups have harmless chain drift practically all the time. For commodity group 'Milk powder' the outcome is natural because prices do not fluctuate at all, and the differences of consumption values and number of commodities between fixed-base and chain-linked GEKS do not change results.

4.1.2 Moderate Contingent Chain Drift

For some group, the chain drift is present for some time periods and it accumulates, while for some, it is not present. We argue that the explanation for differences of fixed-base and chain-linking GEKS is not explained by the strong attrition of commodities (i.e., new commodities). We also see that *IDF* strategy 3 has smaller chain drift compared to strategy 4 (i.e., *IDF* strategy uses 1/13 weights, which '*removes monthly bouncing of consumption*'). We define chain drift as 'Moderate' if the test deviate from unity between the limits (- 0.5 , - 2 %) or (0.5 , 2 %). Moderate chain drift situation covers about 50 % of all *MPIT's*.

Figure 11: Törnqvist for strategies 1 to 4 in region Itä-Suomi and coicop6 = 'Other meat products' from Christmas 2016 to September 2020.

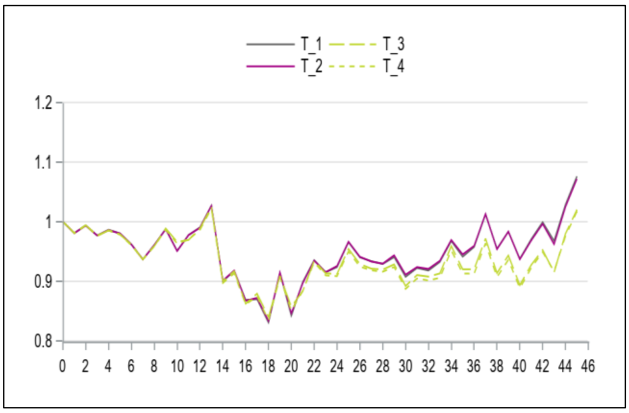


Figure 12: The corresponding MPIT's.

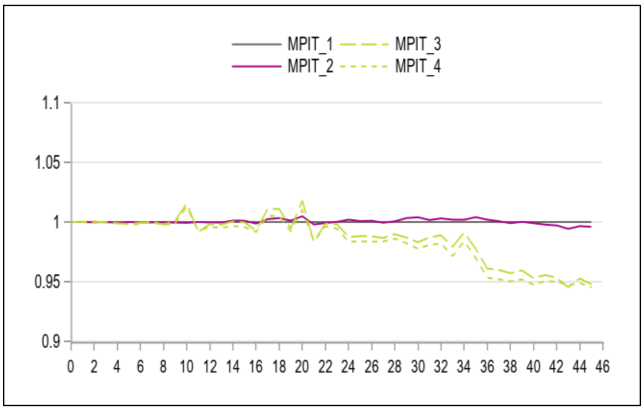


Figure 13: Törnqvist for strategies 1 to 4 in region Etelä-Suomi, coicop6 = 'Pork' from December 2016 to September 2020.

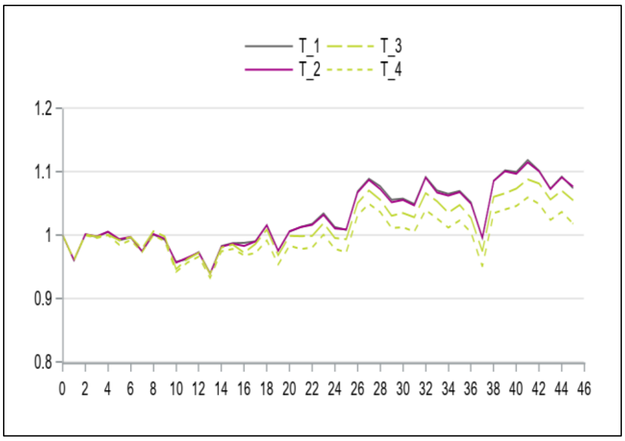
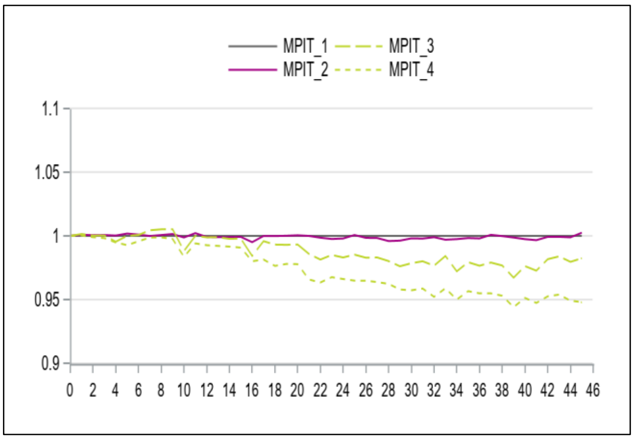


Figure 14: The corresponding MPIT's.



The Figures 11 and 12 tells that chain drift do not exist for time periods (1,…, 20) but starts at about 20th period and cumulates for the rest time periods. This chain drift is caused by the chain strategy of the rolling GEKS (strategies 3 and 4) – new commodities do *not* cause it because the values of consumption (v) and number of commodities (n) are almost equal for coicop6 group 'Other meat products' in region Itä-Suomi all the time (see Figure 15).

Figure 15: Ratio of consumption values and number commodities in region Itä-Suomi and coicop6 = 'Other meat product' from December 2016 to September 2020.

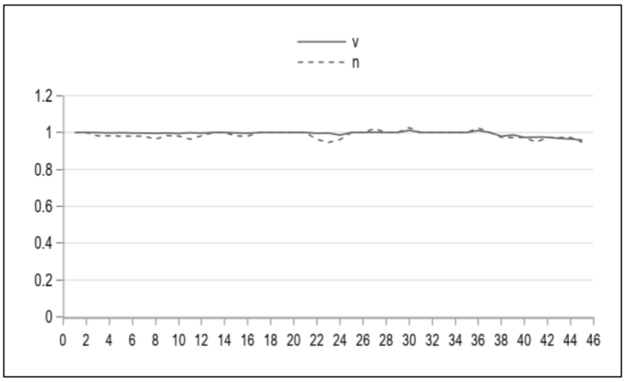
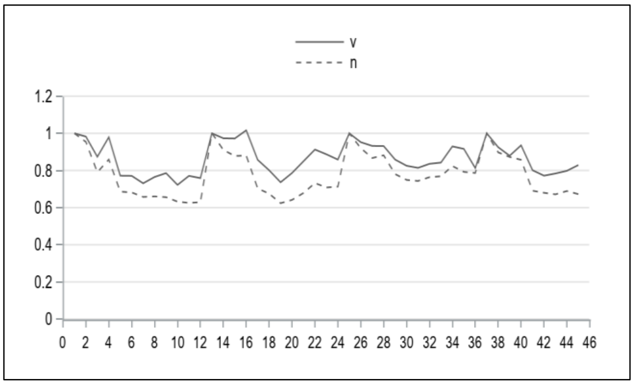


Figure 16: Ratio of consumption values and number commodities in region Etelä-Suomi and coicop6 = 'Pork' from December 2016 to September 2020.



In Figures 13 and 14 the *IDF* (strategy 3) and proper 13-period rolling GEKS (no change of aggregation rule, i.e., the strategy 4) deviates seriously. We think that these deviations are caused by seasonal variation of consumption. In the *IDF*, monthly differences in consumption have been removed by using 1/13-weights. This partly eliminates the true fluctuation or bouncing of monthly consumption but not the chain drift caused by chaining. Figure 15 is important, because it shows that the chain drift appears even if the consumption values and commodities are identical between strategies.

4.1.3 Severe Chain Drift

For some groups, the chain drift is quite large. The chain drift is detected for the commodity groups 'Beef' and 'Pork' quite similarly for all five regions. Figure 18 tells that chain drift occurs in periods 8 and 9 and stays stable from 10 up to 33 periods and after that begins to cumulate. Figure 21 presents ratio of values of consumption and ratio of number of commodities between fixed-base and chain-linking strategies. The share of new commodities varies significantly between time periods 10 to 33 but the price changes for base and chain strategy of GEKS are practically equal (see Figure 21). This is true also for commodity group 'Pork', but now between the periods 16 and 33 (see Figure 22). We define chain drift as 'Serious' if the test deviate from unity more than ± 2 %. We detect serious chain drift situation in about 20 % of all *MPIT's*.

Figure 17: Törnqvist for strategies 1 to 4 in region Uusimaa and coicop6 = 'Beef' from December 2016 To September 2020.

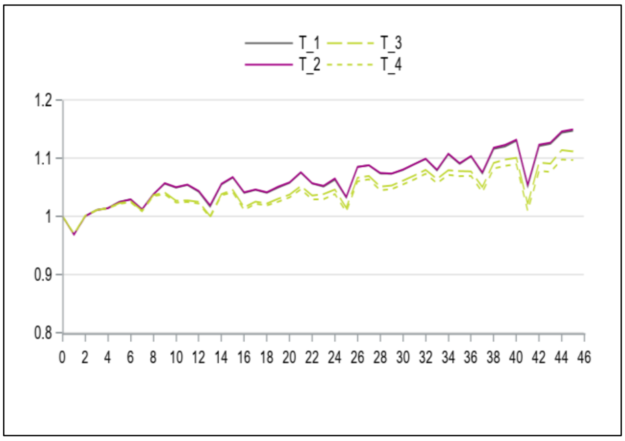


Figure 18: The corresponding MPIT's.

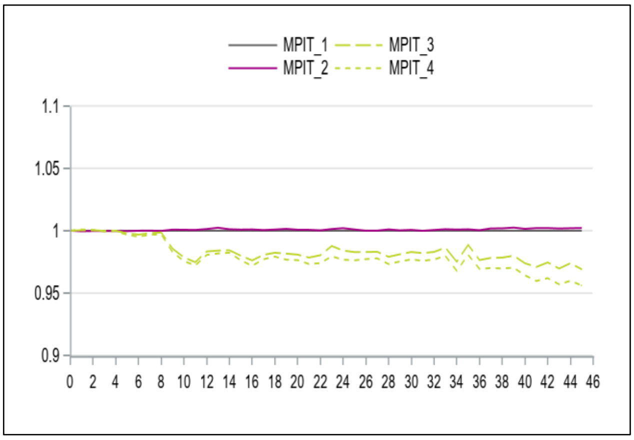


Figure 19: Törnqvist for strategies 1 to 4 in region Uusimaa and 'coicop6 = 'Pork' from December 2016 to September 2020.

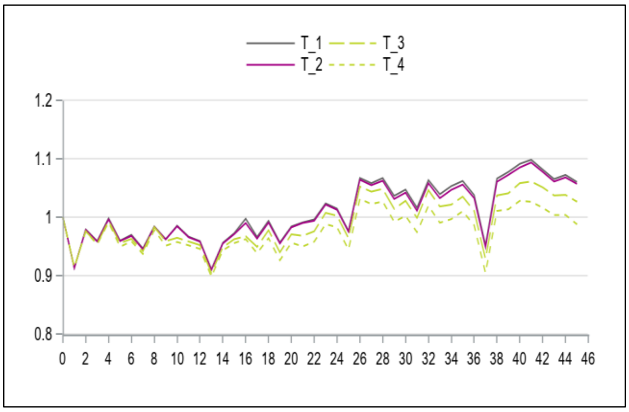


Figure 20: The corresponding MPIT's.

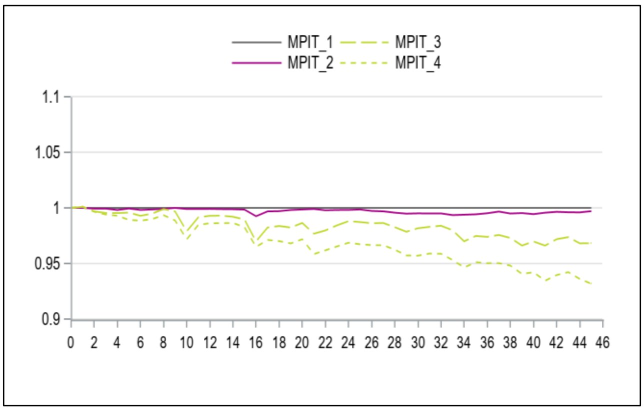


Figure 21: Ratio of consumption values and number commodities in region Uusimaa and coicop6 = 'Beef' from December 2016 to September 2020.

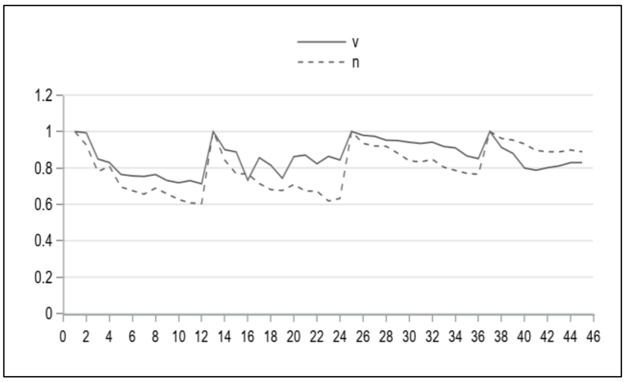
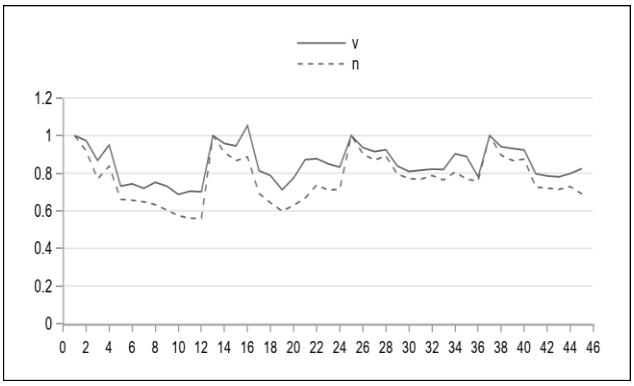


Figure 22: Ratio of consumption values and number commodities in region Uusimaa and coicop6 = 'Pork' from December 2016 to September 2020.



In Figures 21 and 22 we see that every 13th window includes the same data for base and chain strategy of the GEKS. As a conclusion for multilateral fixed-base and chain-linked GEKS-strategies: *If new commodities cause all differences between the strategies of GEKS then these new commodities should be categorized as having strongly declining prices – otherwise the chain drift necessarily explains these differences.*

4.3 The Base GEKS versus Finnish CPI strategy

We have compared empirically the fixed-base and chain-linked strategies of the GEKS. In the *base* and *chain* strategy the price index depends on 12 and 13 *N*-vectors of price and quantity pairs. Quite expectedly, the construction of data based on all price-links for commodities comparable in quality is a hard task - especially for the 13-period rolling GEKS. In our earlier studies (for example in Vartia, Suoperä, Nieminen and Montonen, 2018, 2021) we have marketed a simple construction strategy based on the fixed-base method. In the Finnish CPI strategy we select previous year as the base period and normalize it as average month. The normalization is done by calculating the one-year unit value, and then dividing/normalizing it by 12 to obtain the unit value of an *average month*. It follows, that the average month depends on the all of the previous years' prices and quantities. This simple solution is recommended in Finland for three reasons. *Firstly*, it is a simple multilateral method, and is comparable to the internationally accepted standards. *Secondly*, instead of constructing 12 or 13 price-links for commodities comparable in quality, we construct only one (that is for example *2016* → *2017.m*, *m* = 1, 2, …, 12, see Vartia, Suoperä, Nieminen and Montonen 2021, Circular Error…, pp. 4), thus it simplifies the construction process. *Thirdly*, our strategy effectively solves the problem of seasonal commodities, which is especially serious in Finland. Next, we show differences between our (our strategy here is noted as strategy 5) and 12 period fixed-base strategy of the GEKS (strategy 1) – the figures are very comparable in three excellent index number formulae (i.e., Fisher, Törnqvist and Montgomery-Vartia).

Figure 23: *F*, *T* and *MV* for strategies 1 (solid lines) and 5 (dotted lines) in Uusimaa and commodity group 'Pork' from December 2016 to September 2020.

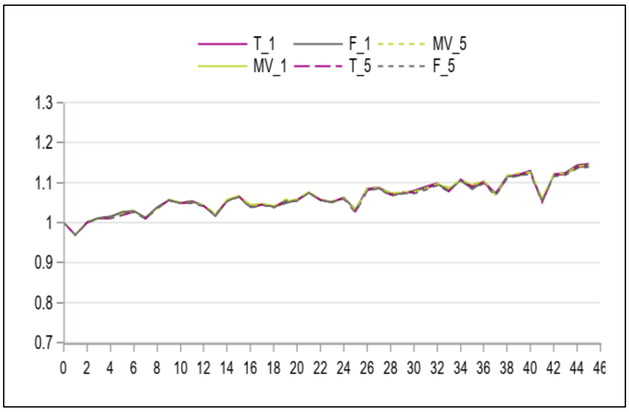


Figure 24: *F*, *T* and *MV* for strategies 1 (solid lines) and 5 (dotted lines) in Etelä-Suomi and group 'Other meat product' from December 2016 to September 2020.

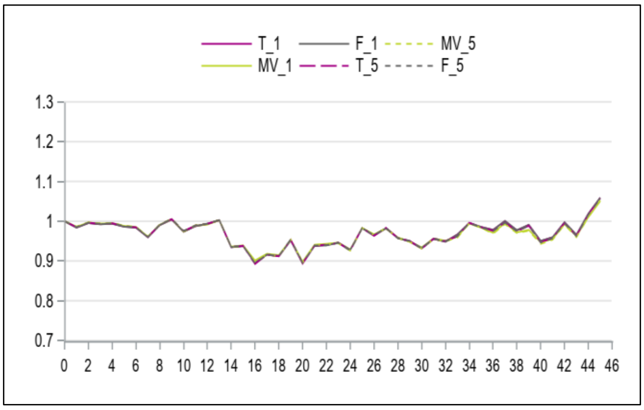


Figure 25: *F*, *T* and *MV* for strategies 1 (solid lines) and 5 (dotted lines) in Itä-Suomi and group 'Fish' from December 2016 to September 2020.

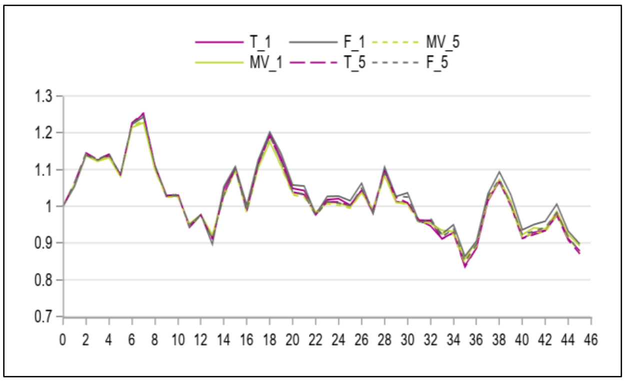
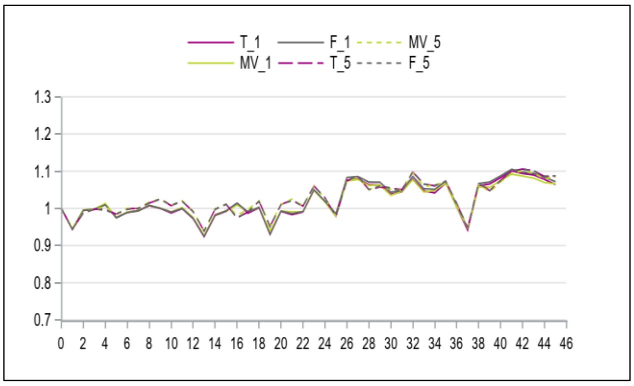


Figure 26: *F*, *T* and *MV* for strategies 1 (solid lines) and 5 (dotted lines) in Pohjois-Suomi and group 'Pork' from December 2016 to September 2020.



The Finnish compilation strategy indeed corresponds to the fixed-base strategy of the GEKS (strategy 1).

4.4 Comparing base and chain Strategy of the GEKS with identical data

Index numbers depend on three things – data, index number formula and construction strategy. We have already shown that the selection of formulas is not a problem at all – all our excellent formulas, including three Diewert-superlative, go ‘hand-in-hand’ all the time. Two problems remain – data and strategy. In this last set of results, we concentrate on differences generated by strategy. For this, we use the 13-period rolling GEKS using the same data as used in calculation of the base GEKS. In that way, we try to mitigate the problem of new commodities that hampers comparisons. Our aim is to get empirical evidence about the differences between multilateral base and chain strategies, that is, about chain drift when the data is exactly the same.

We calculate the Walsh’s MPIT (i.e., eq. (6)) firstly to 13-period rolling GEKS defined in IDF which uses 1/13 weights for multi bilateral indices) and secondly to 13-period rolling GEKS without changing the aggregation rule. We calculate the MPIT’s in five regions for six coicop6 and for 69 kat9 commodity groups. We end up with 30 tests for coicop6 and 345 tests for kat9 groups. The results are presented in following Table.

Table 3: The Walsh’s MPIT between base and chain strategy of GEKS for identical data using Törnqvist formula (see eq. (2a) – (2d) and (3)).

	The Walsh’s MPIT			
	Chain drift larger than ± 2 %		Chain drift larger than ± 5 %	
	Coicop6	KAT9	Coicop6	KAT9
Base vs 13-period rolling GEKS, the IDF-method	19 (63 %)	303 (88 %)	2 (7 %)	195 (57 %)
Base vs pure 13-period rolling GEKS	21 (70 %)	303 (88 %)	5 (17 %)	207 (60 %)

Note: Chain drift larger than +/- 2% refers to situations where the index deviates by more than 2 % at least once during the 2016-2020 period.

Table 3 provides additional evidence, that our earlier observations about the chain drift should be taken seriously. Even with identical data, for base and chain strategies of the GEKS, the multilateral 13-period rolling GEKS strategies includes serious chain-drift, and they should probably not be used for official production of statistics. Our empirical results also show that trade-off between new commodities and chain drift is based on, say, beliefs and not empirical facts. In our view, the chain drift is a bigger problem than the problem of new commodities.

5 Conclusions

This study discusses different ways in which GEKS strategies can be adopted with scanner data. In particular, we apply number of *excellent* index number formulae to the data on food items, test fixed-base and chain-linking strategies with weights and without weights in the aggregation step.

The comparison between the fixed-base and chain-linked GEKS showed that the rolling window GEKS differed sometimes quite substantially, contingent on the analyzed commodity category, from fixed-base GEKS. The reason for differences can be explained only in two ways – by the chain strategy of the rolling GEKS or by new emerging commodities. A simple calculation reveals that if the differences between the base and chain GEKS are caused by the new commodities their prices should decline very seriously. We believe that this is not a plausible explanation. Therefore, the 13-period rolling GEKS necessarily includes a serious chain drift.

With Finnish data, which suffers from strong seasonal consumption patterns, fixed-base GEKS strategy should be preferred over chain-linking and rolling window GEKS, even though the latter works reasonably well over some commodities. This is because, contingently on data, 13-period rolling GEKS will generate a drift, and statistical offices would have a hard time quantifying and explaining its occurrence. The trade-off, of course, is the omission of new commodities from the compilation during the fixed one-year period in fixed-base strategies. However, the consequences of chain drift appear more serious and will certainly arise – in an unexpected fashion creating uncontrollable consequences to the index. The Finnish multilateral fixed-base compilation strategy is extremely simple, has desirable features for official statistics producers in tractability and in ease of implementation, and it produces similar results than the fixed-base GEKS strategies.
The choice of formula makes little difference in our case, any of the excellent index number formulae will provide reasonable price indices. However, the aggregation scheme does make a difference, and weights should be incorporated to fixed-base strategies if they are available in the data. This is in line with the recommendations in Pursiainen (2005) which stresses the importance of the idea that same formula should be maintained in all aggregation steps.

Next analyses should still focus more closely on tracking down exactly the role of new commodities, consumption spikes, and seasonality in generating the drift. This remains an open issue, even though in the current data, the role of new commodities is very limited, as we have focused on a relatively stable set of food items. We recommend a cautious and pragmatic approach to the compilation of CPIs, even with the possibilities offered with new scanner data.

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